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ANALYSIS OF $B^\pm \rightarrow K^+ K^- K^\pm$ DECAYS

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Rare B^\pm decays to three charged kaons are analysed. The weak decay amplitudes are derived in the QCD factorization approach. The strong final state interactions between pairs of kaons are described using the kaon scalar and vector form factors. The scalar form factors at low $K^+ K^-$ effective mass distributions are constrained by chiral symmetry and are related to the coupled channel meson-meson amplitudes describing all the transitions between three channels consisting of two kaons, two pions and four pions. The vector form factors are fitted to the data on $e^+ e^-$ collisions. The model results are compared with the Belle and BaBar data.

Keywords: rare B decays; final state interactions; CP asymmetry.

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1. Theoretical model

We study rare charged B meson decays into three charged kaons. In search for direct CP violation the strong interactions between kaons must be included. Partial wave analysis of the decay amplitudes helps in investigation of the density distributions of the Dalitz diagrams. The QCD quasi-twobody factorization approach for a limited range of the effective $K^+ K^-$ masses less than about 1.8 GeV is used.

The amplitude for the B^- decay into three charged kaons reads:

$$\begin{aligned} \langle K^-(p_1) K^+(p_2) K^-(p_3) | H | B^- \rangle = & \frac{G_F}{\sqrt{2}} \left\{ \sqrt{\frac{1}{2}} \chi f_K (M_B^2 - s_{23}) F_0^{B \rightarrow (K^+ K^-)_S}(m_K^2) u \right. \\ & \Gamma_2^{n*}(s_{23}) + \frac{2B_0}{m_b - m_s} (M_B^2 - m_K^2) F_0^{BK}(s_{23}) v \Gamma_2^{s*}(s_{23}) + 4 \vec{p}_1 \cdot \vec{p}_2 \left[\frac{f_K}{f_\rho} A_0^{B\rho}(m_K^2) u \right. \\ & \left. \left. F_u^{K^+ K^-}(s_{23}) - F_1^{BK}(s_{23}) (w_u F_u^{K^+ K^-}(s_{23}) + w_d F_d^{K^+ K^-}(s_{23}) + w_s F_s^{K^+ K^-}(s_{23})) \right] \right\}, \end{aligned} \quad (1)$$

where G_F is the Fermi coupling constant, f_K and f_ρ are the kaon and ρ meson decay constants, M_B , m_K , m_b , m_s , m_u and m_d are the masses of the B meson, kaon, b -quark, strange quark, up- and down quarks, respectively; s_{23} is the $K^+(p_2) K^-(p_3)$ effective mass squared, \vec{p}_1 and \vec{p}_2 are the kaon 1 and kaon 2 momenta in the center of mass system of the kaons 2 and 3. The functions Γ_2^n and Γ_2^s are the kaon

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nonstrange and strange scalar form factors, $F_u^{K^+K^-}$, $F_d^{K^+K^-}$ and $F_s^{K^+K^-}$ denote three types of the kaon vector form factors, $F_0^{B \rightarrow (K^+K^-)_S}$ is the form factor of the transition from the B meson to the K^+K^- pair in the S -wave, χ is the constant related to the decay of the $(K^+K^-)_S$ state into two kaons, F_0^{BK} , F_1^{BK} and $A_0^{B\rho}$ are the $B \rightarrow K$ and $B \rightarrow \rho$ transition form factors. The constant $B_0 = m_\pi^2/(m_u + m_d)$, m_π being the pion mass. Other symbols are defined in terms of Wilson's coefficients a_j and the products $\Lambda_u = V_{ub}V_{us}^*$, $\Lambda_c = V_{cb}V_{cs}^*$, V_{ij} being the CKM quark-mixing matrix elements:

$$u = \Lambda_u[a_1 + a_4^u + a_{10}^u - (a_6^u + a_8^u)r_\chi] + \Lambda_c[a_4^c + a_{10}^c - (a_6^c + a_8^c)r_\chi], \quad (2)$$

$$v = \Lambda_u(-a_6^u + \frac{1}{2}a_8^u) + \Lambda_c(-a_6^c + \frac{1}{2}a_8^c), \quad r_\chi = \frac{2m_K^2}{(m_b + m_u)(m_u + m_s)} \quad (3)$$

$$w_u = \Lambda_u a_2 + (\Lambda_u + \Lambda_c)(a_3 + a_5 + a_7 + a_9), \quad w_d = (\Lambda_u + \Lambda_c)[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)], \quad (4)$$

$$w_s = \Lambda_u[a_3 + a_4^u + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}^u)] + \Lambda_c[a_3 + a_4^c + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}^c)]. \quad (5)$$

The decay amplitude given by Eq. (1) consists of two parts: the P -wave part proportional to the product $\vec{p}_1 \cdot \vec{p}_2$ and the preceding term being the S -wave part. In the final state of the decay reaction we take into account the elastic kaon-kaon S -wave interactions and other transitions like the K^+K^- annihilations into systems consisting of two and four pions. Thus a system of three coupled channels: $\pi\pi$, $\bar{K}K$ and 4π (effective $\sigma\sigma$), labelled by 1,2 and 3, is considered. A set of the 3x3 transition amplitudes T is taken from a unitary model developed in Ref. 1. At first the production functions R_j are introduced as follows:

$$R_j(E) = (\alpha_j + \tau_j E + \omega_j E^2)/(1 + CE^4), \quad E \equiv m_{K^+K^-}, \quad (6)$$

where $\alpha_j, \tau_j, \omega_j$ and C are constant parameters. Then the three scalar form factors, written in the compact matrix form Γ are expressed by

$$\Gamma^* = R + TGR, \quad (7)$$

where G is the matrix of the channel Green functions. The coefficients α_j, τ_j and ω_j are constrained by the values of the form factors, calculated in the framework of the chiral model of Ref. 2, using the numerical results of the lattice QCD³. The parameter C , which controls the high energy behaviour of R , can be fixed while fitting the data of the BaBar⁴ and Belle⁵ collaborations. The present model of the scalar form factors satisfies the unitarity conditions.

The three vector form factors $F_q^{K^+K^-}$ for $q = u, d, s$ defined as

$$\langle K^+(p_2)K^-(p_3)|\bar{q}\gamma_\mu q|0 \rangle = (p_2 - p_3)_\mu F_q^{K^+K^-}(s_{23}) \quad (8)$$

enter into the P -wave amplitude. These functions contain contributions from eight vector mesons $\rho(770)$, $\rho(1450)$, $\rho(1700)$, $\omega(782)$, $\omega(1420)$, $\omega(1650)$, $\phi(1020)$ and $\phi(1680)$. Here the model of Ref. 6 has been used in their parametrization.

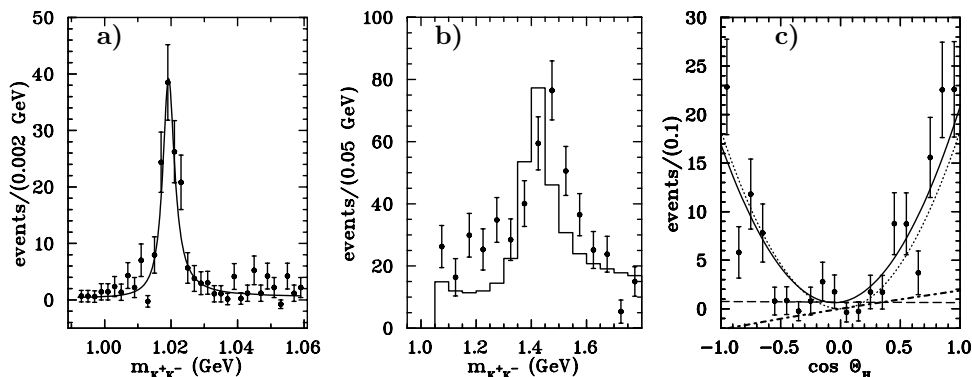


Fig. 1. The K^+K^- effective mass distributions from the fit to the Belle experimental data in the $\phi(1020)$ range (a) and between 1.05 GeV and 1.75 GeV (b). The theoretical results are shown as the solid line in (a) and as the histogram in (b). Helicity angle distributions for events in the K^+K^- effective mass up to 1.05 GeV (c). The dashed line represents the S -wave contribution of our model, the dotted line - that of the P -wave, the dot-dashed - that of the interference term and the solid line corresponds to the sum of these contributions.

2. Results

Preliminary results of the model fits to the data of the Belle ⁵ Collaboration are shown in Fig. 1. The scalar resonance $f_0(980)$ leads to the threshold enhancement of the S -wave amplitude. The K^+K^- structure seen near 1.5 GeV can be attributed to another scalar resonance. Sizable helicity angle asymmetry appears in the $\phi(1020)$ range. A potentially large CP asymmetry can be discovered in the mass spectrum dominated by the S -wave. However, new experimental analyses of data with better statistics are needed. The data already exist! For example, the Belle Collaboration has now five times larger data sample than that used in their publication ⁵. Future results from super B factories will also be very useful.

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